Attitude?

Why? (5)

Governing equations?

Rotation parameterization?

Disturbance torques?

\[ \frac{d(H)}{dt} = \frac{d(I\dot{\omega})}{dt} = \vec{T}_{\text{disturb}} \]

\[ \hat{q} = q_0 + iq_1 + jq_2 + kq_3 \]

(4-D complex number)
How to stabilize the spacecraft with respect to disturbance torques?
How to stabilize the spacecraft with respect to disturbance torques?

1. **Passive attitude control**

\[
\frac{d(\vec{H})}{dt} = \frac{d(\vec{I}\,\vec{\omega})}{dt} = \vec{T}_{\text{disturb}}
\]

**EXPLOIT!**
How to stabilize the spacecraft with respect to disturbance torques?

2. **Active attitude control**
   (determination + control)

\[
\frac{d(\vec{H})}{dt} = \frac{d(\vec{I} \vec{\omega})}{dt} = \vec{T}_{\text{disturb}} + \vec{T}_{\text{control}}
\]
How to stabilize the spacecraft with respect to disturbance torques?

2. **Active attitude control**
   (determination + control)

\[
\frac{d(\vec{H})}{dt} = \frac{d(\vec{I} \omega)}{dt} = \vec{T}_{\text{disturb}} + \vec{T}_{\text{control}}
\]

**APPLY!**
ADCS: **Attitude** Determination and Control System

AOCS: **Attitude** and **Orbit** Control System
A **nominal orbit** is defined for the satellite. This is the orbit the satellite should maintain.

The orbit is controlled by applying forces to the satellite. Forces are applied by $\Delta V$ actuators.

- **Launch vehicle**
  - $\sim 10^6 \text{ N, 10 minutes, } \Delta V \sim 10 \text{ km/s}$

- **Apogee/perigee motors**
  - $\sim 10^4 \text{ N, 1 minute, } \Delta V \sim 2 \text{ km/s}$

- **Station keeping**
  - $\sim [10^{-3}, 10^1] \text{ N, intermittent, } \Delta V \sim 0.35 \text{ km/s (7 years)}$
Braking maneuver (Venus Express)

Apogee motor for orbit circularization (Eutelsat)

Meteosat: 6 thrusters for orbit maintenance
A nominal attitude is defined for the satellite. This is the attitude that the satellite should ideally maintain.

The satellite attitude is controlled by applying torques to the satellite. Torques are applied by attitude actuators.
Attitude Control vs. Orbit Control

How many DOFs does a (rigid) satellite possess?

Trajectory dynamics — Orbit control
- Linear momentum
- Motion of the center of mass

Attitude dynamics — Attitude control
- Angular momentum
- Motion relative to the center of mass

More details in *Astrodynamics* (AERO0024)

Focus in this lecture

??
Orbit and Attitude Are Interdependent

Examples:

1. In LEO, the attitude will affect the atmospheric drag which will affect the orbit

2. The orbit determines the spacecraft position which determines both the atmospheric density and the magnetic field strength, which will, in turn, affect the attitude

But this dynamic coupling is often ignored, and the time history of the spacecraft position is assumed to be known and to be an input for ADCS
Attitude Definition

Angular orientation of a **body-fixed coordinate frame** with respect to an **external frame**
Body Frames

Usually at the center of mass of the spacecraft

Spacecraft designers normally selects a principal axis coordinate frame. If not, specifications limit the allowable magnitude of the products of inertia in the defined frame

\[
I = \begin{pmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{pmatrix}
\]

Proba 2 (130 kg)
External: Earth Centred Inertial (ECI)

Origin at the Earth’s center of mass and does not rotate with the planet

X-axis: intersection between the ecliptic and equatorial planes at a particular moment (e.g., 1\textsuperscript{st} of January 12pm)

Z-axis: Earth’s center-North pole in the northerly direction

Y-axis: right-handed set

Is it an inertial frame?
Visualization: Attitude Sphere
ESEO, solar panels: accuracy: [1° -5°]

Envisat, Earth observation:
accuracy: ~0.01°
stability: ~0.001°

HST, astronomy: accuracy: ~0.1”
stability: ~0.01”, 1” = 1°/3600
Structural flexibility is a real issue
Relate external torques to angular velocity to understand the influence on attitude dynamics.
**Key Concept: Angular Momentum**

Fundamental quantity in rotational dynamics

Moment of the linear momentum about a defined origin

\[ \vec{H} = \vec{r}_i \wedge m_i \vec{v}_i \]

The angular momentum of a particle referred to an inertially fixed point is only changed if the forces on it have a moment \( M \) about this fixed point

\[ \frac{d(\vec{H})}{dt} = \vec{T} \]
Key Concept: Angular Momentum

In the absence of external torque, the angular momentum is preserved

\[
\frac{d(\vec{H})}{dt} = 0
\]

What does happen to the angular momentum when the spacecraft deploys its solar panels?
Time Differentiation in a Rotating Frame

\[ \begin{align*}
\vec{r}_i &= \vec{R} + \vec{\rho}_i \\
\vec{v}_i &= \frac{d\vec{r}_i}{dt} = \frac{d\vec{R}}{dt} + \left( \frac{d\vec{\rho}_i}{dt} \right)_{\text{body}} + \vec{\omega} \wedge \vec{\rho}_i \\
\vec{a}_i &= \frac{d^2\vec{R}}{dt^2} + \left( \frac{d^2\vec{\rho}_i}{dt^2} \right)_{\text{body}} + 2\vec{\omega} \wedge \left( \frac{d\vec{\rho}_i}{dt} \right)_{\text{body}} + \frac{d\vec{\omega}}{dt} \wedge \vec{\rho}_i + \vec{\omega} \wedge \left( \vec{\omega} \wedge \vec{\rho}_i \right)
\end{align*} \]
Total Angular Momentum

\[ \vec{H}_{\text{total}} = \vec{H}_t = \sum_i \vec{r}_i \wedge m_i \vec{v}_i \]

\[ \vec{r}_i = \vec{R} + \vec{\rho}_i \]

\[ \vec{v}_i = \frac{d\vec{r}_i}{dt} = \frac{d\vec{R}}{dt} + \vec{\omega} \wedge \vec{\rho}_i \]

Rigid body: \((d\rho_i/dt)_{\text{body}} = 0\)

\[ \vec{H}_t = \sum_i m_i \vec{R} \wedge \frac{d\vec{R}}{dt} + \sum_i m_i \vec{\rho}_i \wedge (\vec{\omega} \wedge \vec{\rho}_i) \]

\[ \vec{I} = \sum_i m_i (\rho_i^2 \vec{E}_3 - \vec{\rho}_i \vec{\rho}_i) \]

\[ \vec{a} \wedge (\vec{b} \wedge \vec{c}) = \]

\[ \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) = \]

\[ \vec{b}(\vec{a} \cdot \vec{c}) - \vec{b}(\vec{c} \cdot \vec{a}^T) = \vec{b}(\vec{E}_3 \vec{a} \cdot \vec{c} - \vec{c} \vec{a}^T) \]

\[ \vec{H}_t = \sum_i m_i \vec{R} \wedge \vec{V} + \vec{I} \vec{\omega} \]
The angular momentum is equivalent to the linear momentum for rotational dynamics \((m \Rightarrow I, v \Rightarrow \omega)\)

For a rigid spacecraft, the spin angular momentum can be decoupled from the orbital angular momentum

Choose principal axes of inertia! In practice, asymmetries and misalignments \(\Rightarrow\) coupling \(\Rightarrow\) unwanted disturbances to be removed by the control system
**External Torques**

\[ \vec{T}_i = \vec{\rho}_i \wedge \vec{F}_i \]

A force applied to the spacecraft produces a torque about the center of mass

\[ \vec{T}_{total} = \vec{T} = \sum_i \vec{\rho}_i \wedge \vec{F}_i = \sum_i \vec{\rho}_i \wedge m_i \frac{d^2 \vec{r}_i}{dt^2} \]

\[ \vec{a}_i = \frac{d^2 \vec{R}}{dt^2} + \left( \frac{d^2 \vec{\rho}_i}{dt^2} \right)_{body} + 2\vec{\omega} \wedge \left( \frac{d \vec{\rho}_i}{dt} \right)_{body} + \frac{d\vec{\omega}}{dt} \wedge \vec{\rho}_i + \vec{\omega} \wedge (\vec{\omega} \wedge \vec{\rho}_i) \]

\[ \vec{H}_{s/c} = \vec{I} \vec{\omega} \]

\[ \vec{T} = \frac{d\vec{H}}{dt} = \left( \frac{d\vec{H}}{dt} \right)_{s/c} + \vec{\omega} \wedge \vec{H}_{s/c} = \vec{I} \left( \frac{d\vec{\omega}}{dt} \right)_{body} + \vec{\omega} \wedge \vec{I} \vec{\omega} \]

Fundamental equation for attitude dynamics

(Newton’s second law for rotating rigid bodies)
Equation Governing Attitude Dynamics

\[ \vec{T} \leftrightarrow \vec{\omega} \]

\[ \vec{T} = \vec{I} \left( \frac{d{\vec{\omega}}}{dt} \right)_{\text{body}} + \vec{\omega} \wedge \vec{I} \vec{\omega} \]

Principal axes

Gyroscopic coupling

\[ \dot{H}_x = I_x \dot{\omega}_x = T_x + (I_y - I_z)\omega_y \omega_z \]
\[ \dot{H}_y = I_y \dot{\omega}_y = T_y + (I_z - I_x)\omega_x \omega_z \]
\[ \dot{H}_z = I_z \dot{\omega}_z = T_z + (I_x - I_y)\omega_x \omega_y \]

Nonlinear equations with no general solution \(\Rightarrow\) computer simulations
Spinning Rigid Spacecraft

\[ \omega_x, \omega_y \ll \omega_z = \Omega \] Spin

Free response

\[
\begin{align*}
\dot{\omega}_x &= \frac{I_z - I_y}{I_x} \Omega \omega_y \\
\dot{\omega}_y &= \frac{I_z - I_x}{I_y} \Omega \omega_x \\
\dot{\omega}_z &= 0
\end{align*}
\]

Nutation frequency

\[ \Omega_n^2 = \frac{I_z - I_y}{I_x} \frac{I_z - I_x}{I_y} \Omega^2 \]
Spinning Rigid Spacecraft

What does happen if the spin axis inertia $I_z$ is intermediate between $I_x$ and $I_y$?

The nutation frequency is imaginary. Any perturbing torque will result in the growth of the nutation angle until the body is spinning about either the maximum or minimum inertia axis, depending on initial conditions.

A rigid body can rotate about its extreme inertia axis, but not the intermediate axis.
$I_1 > I_2 > I_3$
Two Spinning Eggs: Raw and Hard-Boiled
Spinning Flexible Spacecraft

Flexibility $\Rightarrow$ energy dissipation $\Rightarrow$ heat. Because total energy is conserved, heat must be derived from the rotational kinetic energy

$$E_{rot} = \frac{1}{2} I \omega^2, H = I \omega, E_{rot} = \frac{1}{2} \frac{H^2}{I}$$

H must be constant, if no permanent deformation $\Rightarrow$ moment of inertia must increase $\Rightarrow$ the spin axis must shift in body coordinates (because H must be constant, the spin axis is fixed in inertial coordinates)

A real (therefore flexible) body can spin stably only about the axis of maximum moment of inertia
Two Spinning Eggs: Raw and Hard-Boiled

The hard-boiled egg can be assumed to be a rigid body
⇒ It can rotate around its minimum-inertia axis

The raw egg has a very rapid internal energy dissipation
⇒ It falls into a flat spin; it rotates at a slower rate about the maximum-inertia principal axis
Explorer 1 Spacecraft

What happened?

Spin stabilized about its long axis (750 rpm)

Four flexible wire antennas for communication

First US satellite and first spacecraft to detect Van Allen radiation belts (1st February 1958)
To the surprise of mission experts, satellite Explorer 1 changed rotation axis after launch. The elongated body of the spacecraft had been supposed to spin about its long (least-inertia) axis but refused to do so, and instead started precessing due to energy dissipation from flexible structural elements.

This motivated the first further development of the Eulerian theory of rigid body dynamics (after nearly 200 years) to address dissipation.
Rotational Dynamics is Complex...
If a hard-boiled egg is spun sufficiently rapidly on a table with its axis of symmetry horizontal, this axis will rise from the horizontal to the vertical. (A raw egg, by contrast, when similarly spun, will not rise.)
Can a spinning egg really jump?

BY T. Mitsui¹, K. Aihara², C. Terayama¹, H. Kobayashi¹
AND Y. Shimomura¹,*

Figure 5. A sequence of snapshots of a hard-boiled egg spinning on a table. The hard-boiled egg, whose weight, major radius and minor one are, respectively, 67.3 g, 30.2 and 22.6 mm, is spun on a table with its axis of symmetry initially horizontal at a spin rate of 1800 r.p.m. by an electric motor with the same support of the body as shown in figure 1. The coefficient of dynamic friction at a point of contact between the egg and the table is 0.17.
Rotational Dynamics is Complex…

Does it violate the conservation of angular momentum?
Attitude？

Why？(5)

Governing equations？

Rotation parameterization？
Rotations

How many angles do we need to represent the spatial orientation of any frame of the space with respect to a reference frame?

Any rotation can be viewed as a succession of three planar rotation about three different axes

⇒ Euler angles (precession, nutation, spin)

⇒ Bryant angles (roll, yaw, pitch)
Euler Angles

First rotation:
\[ R_1(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

Second rotation:
\[ R_2(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \]

Third rotation:
\[ R_3(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ R = R_1 R_2 R_3 = \begin{bmatrix} \cos \phi \cos \psi - \sin \phi \cos \theta \sin \psi & -\cos \psi \sin \phi - \cos \theta \sin \psi \cos \phi & \sin \psi \sin \theta \\ \cos \phi \sin \psi + \cos \theta \cos \psi \sin \phi & -\sin \phi \sin \psi + \cos \theta \cos \psi \cos \phi & -\cos \psi \sin \theta \\ \sin \theta \sin \phi & \sin \theta \cos \phi & \cos \theta \end{bmatrix} \]

\[ R = \begin{bmatrix} \cos (\phi \pm \psi) & -\sin (\phi \pm \psi) & 0 \\ \sin (\phi \pm \psi) & \cos (\phi \pm \psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

Singularity if \( \theta = 0, \pi \)
Singularity-Free Behavior

All parameterizations with 3 parameters have a singularity

To remove singularity, a fourth (redundant) parameter can be introduced

The direction cosine matrix introduces 9 parameters

\[ R = \begin{bmatrix}
\cos(\vec{i}, \vec{i}') & \cos(\vec{i}, \vec{j}') & \cos(\vec{i}, \vec{k}') \\
\cos(\vec{j}, \vec{i}') & \cos(\vec{j}, \vec{j}') & \cos(\vec{j}, \vec{k}') \\
\cos(\vec{k}, \vec{i}') & \cos(\vec{k}, \vec{j}') & \cos(\vec{k}, \vec{k}')
\end{bmatrix} \]
Euler’s Theorem

The Orientation of a body is uniquely specified by a vector giving the direction of a body axis and a scalar specifying a rotation angle about the axis.
The elements of a quaternion can be understood as a rotation axis $n$ and a rotation angle $\varphi$ about this axis.

$$q_1 = n_1 \sin \left( \frac{\varphi}{2} \right) \quad q_3 = n_3 \sin \left( \frac{\varphi}{2} \right)$$

$$q_2 = n_2 \sin \left( \frac{\varphi}{2} \right) \quad q_4 = \cos \left( \frac{\varphi}{2} \right)$$
Quaternions

For a given axis and angle, one can easily construct the corresponding quaternion, and conversely, for a given quaternion one can easily read off the axis and the angle.

\[
R = \begin{bmatrix}
q_4^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_4q_3) & 2(q_1q_3 + q_4q_2) \\
2(q_1q_2 + q_4q_3) & q_4^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_4q_1) \\
2(q_1q_3 - q_4q_2) & 2(q_2q_3 + q_4q_1) & q_4^2 - q_1^2 - q_2^2 + q_3^2
\end{bmatrix}
\]
Quaternions: Advantages

+ No trigonometric functions (faster computations)
- No singularity

- Physical interpretation

Tomb Raider

Satellites
Rotations Are Not Commutative

\[ R_1 R_2 \neq R_2 R_1 \]
Attitude?

Why? (5)

Governing equations?

Rotation parameterization?

Disturbance torques?
Disturbance Torques

Torques

External
- Aerodynamic
- Gravity-gradient
- SRP
- Magnetic
- Jettisoned parts
- Venting (relief valves)

Internal
- Solar panel deployment
- Instrument motion
- Sloshing
- Etc.

Affect the total angular momentum

Total angular momentum conserved but influence spacecraft orientation
Aerodynamic Torque

The drag force produces a disturbance torque on the spacecraft due to any offset that exists between the aerodynamic center of pressure and the center of mass.

\[ \vec{T} = \vec{r}_{cp} \wedge \vec{F}_a \]

\[ \vec{F}_a = \frac{1}{2} \rho V^2 S C_D \frac{V}{V} \]

\sim 2 \text{ for free-molecular flow}
Aerodynamic Torque

Above 200km altitude, the mean free path is significantly greater than the dimensions of most space vehicles

- Aerodynamics must be based upon free molecular flow
- Heat exchange solely due to radiation (no convection)

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>$\lambda_0$ (m)</th>
<th>Altitude (km)</th>
<th>$\lambda_0$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.142</td>
<td>300</td>
<td>$2.6 \times 10^3$</td>
</tr>
<tr>
<td>120</td>
<td>3.31</td>
<td>400</td>
<td>$16 \times 10^3$</td>
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<tr>
<td>140</td>
<td>18</td>
<td>500</td>
<td>$77 \times 10^3$</td>
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<tr>
<td>160</td>
<td>53</td>
<td>600</td>
<td>$280 \times 10^3$</td>
</tr>
<tr>
<td>180</td>
<td>120</td>
<td>700</td>
<td>$730 \times 10^3$</td>
</tr>
<tr>
<td>200</td>
<td>240</td>
<td>800</td>
<td>$1400 \times 10^3$</td>
</tr>
</tbody>
</table>
July 15, 2000: a strong solar flare heated the Earth’s atmosphere, increasing the air density to a value 100 times greater than that for which its ADCS had been designed to cope. The magnetorquers were unable to compensate and the satellite was lost.

\[
\rho = \rho_{SL} \exp\left(\frac{-gM_i}{R^* T} Z\right)
\]

Simplified expression. Otherwise difficult to evaluate.
\[ S = 5 \text{ m}^2 \]
\[ C_D = 2 \]
\[ h = 400 \text{ km} \Rightarrow \rho = 4 \times 10^{-12} \text{ kg/m}^3 \]
\[ r_{CP} = 0.01 \text{ m} \]

For \( I = 1000 \text{ kg.m}^2 \)

\[ T = \frac{dH}{dt} = I \frac{d\omega}{dt} = I \frac{d^2 \theta}{dt^2} \]

\[ \theta(0) = 0 \]
\[ \dot{\theta}(0) = 0 \]

\[ T = 1.2 \times 10^{-5} \text{ Nm} \]

\[ \theta(t) = \frac{T}{I} t^2 = 1.2 \times 10^{-8} \text{ s}^{-2} t^2 \]

Drift of 57° after 10000s (2 orbits) !!!

Unacceptable.
Gravity-Gradient Torque

Gravitational fields decrease with distance from the center of the planet ⇒ a spacecraft in orbit experiences a stronger attraction on its “lower” side than its “upper” side.

3 equilibrium positions: 2 stable and 1 unstable
Gravity-Gradient Torque

\[ d\vec{F} = -\mu \frac{\vec{\rho} + \vec{r}}{|\vec{\rho} + \vec{r}|^3} dm \]

\( \mu = \) gravitational constant

\[ d\vec{T} = \vec{\rho} \wedge d\vec{F} \Rightarrow \vec{T} = \int \vec{\rho} \wedge \left( -\mu \frac{\vec{\rho} + \vec{r}}{|\vec{\rho} + \vec{r}|^3} \right) dm \]
Gravity-Gradient Torque

\[ \hat{\rho} \ll \hat{r} \Rightarrow \frac{1}{|\hat{\rho} + \hat{r}|^3} \approx r^{-3} \left[ 1 - 3 \frac{\hat{r}}{r^2} \hat{\rho} \right] \]

\[ \vec{T} = \int \hat{\rho} \wedge \left[ -\frac{\mu}{r^3} \left( 1 - 3 \frac{\hat{r}}{r^2} \hat{\rho} \right) \hat{r} \right] dm \]

\[ \int \hat{\rho} dm = 0, \quad \hat{r} = \frac{\hat{r}}{r} \]

\[ \vec{T} = \frac{3\mu}{r^3} \hat{r} \wedge (\int \hat{\rho} \hat{\rho}^T dm) \hat{r} \]

\[ \vec{T} = \frac{3\mu}{r^3} \hat{r} \wedge \hat{I} \hat{r} \]

⊥ to Earth-spacecraft vector
Gravity-Gradient Torque

- Local vertical

- Yaw $\psi$

- Roll $\phi$

- Pitch $\theta$

$X: // \vec{v}$

$Y$

$Z: // \vec{r}$
Gravity-Gradient Torque

In the spacecraft body frame, the unit vector to the spacecraft from the planet is

\[ \hat{r} = (-\sin \theta, \sin \phi, 1 - \sin^2 \theta - \sin^2 \phi)^T \]

For a 3-axis stabilized spacecraft, deviations are necessarily small

\[ \hat{r} \approx (-\theta, \phi, 1)^T \]
Gravity-Gradient Torque

\[ \vec{T} = \frac{3\mu}{r^3} \hat{r} \wedge \vec{I} \hat{r} \]

\[ \hat{r} \approx (-\theta, \phi, 1)^T \]

The satellite axes coincide with the principal axes

\[ \vec{T} = [T_x \quad T_y \quad T_z]^T = 3n^2 \begin{bmatrix} (I_z - I_y)\phi & (I_z - I_x)\theta & 0 \end{bmatrix}^T \]

The yaw angle \( \psi \) does not influence the gravity-gradient torque (intuitively reasonable because yaw represents rotation around the local vertical)

The torque magnitude depends upon the difference between principal moments. Spacecraft that are long and thin are more affected
Gravity-Gradient Torque: Example

Altitude = 700km
Inertia moment difference = 30 kg.m²
θ = 1deg

\[ T = \frac{3 \times 3.986 \times 10^{14} \times 30 \times 0.0175}{(7.078 \times 10^6)^3} = 1.8 \times 10^{-6} \text{ N.m} \]
Solar Radiation Pressure Torque

Solar radiation pressure produces a disturbance torque on the spacecraft, which depends upon the distance from the sun. It is independent of spacecraft position and velocity and is perpendicular to the sun line.

Solar radiation is NOT related to solar wind, which is a continuous stream of particles emanating from the sun. The momentum flux in the solar wind is small compared with that due to solar radiation.

| Solar radiation (photons) | ≠ | Solar wind (particles) |
Solar Radiation Pressure Torque

\[
\vec{T} = \vec{r} \wedge \vec{F}_s
\]

Projected area normal to sun vector

Vector from body center of mass to center of solar pressure

Surface reflectivity

1360 W/m² / 3x10⁸ m/s

\[
\vec{F}_s = (1 + K) \rho_s A_p
\]

\[
A_p = 5 \text{ m}^2, K = 0.5, r = 0.1 \text{ m}
\]

\[
T = 3.5 \times 10^{-6} \text{ N.m}
\]
Magnetic Torque

The magnetic field generated by a spacecraft interacts with the local field from the Earth

\[ \vec{T} = \vec{M} \land \vec{B} \]

Spacecraft magnetic dipole moment (current loops+residual magnetic moment). E.g., residual moment in the magnetorquers of PROBA 2 = 0.03 A.m²

Earth magnetic field vector in spacecraft coordinates (proportional to \(1/r^3\), r: radius vector to the spacecraft)

\[ B(700 \text{ km}) = 2M_{\text{earth}} / R^3 = 2 \times 7.96 \times 10^{15} / (6378 + 700)^3 \approx 4.5 \times 10^{-5} \text{ Tesla} \]

\[ M(\text{small s/c}) \approx 0.1 \text{ Ampere.m}^2 \]

\[ T = 4.5 \times 10^{-6} \text{ N.m} \]
The Earth's Magnetic Field

North Magnetic Pole

Geographic North Pole

11.5°

Geographic South Pole

South Magnetic Pole

*The Earth's North Magnetic Pole is in fact a south pole. North poles on compasses point towards it. Notice that the compass needle in the picture has the white (south) tip pointing north, and the field line arrows point from south to north.

Larger versions of this image are available: contact peter.reid@ed.ac.uk

Peter Reid, 2007
Total magnetic field intensity at Earth’s surface in Gauss (=0.0001 Tesla) in 1965
## Comparison

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>Dependence on the distance to the Earth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerodynamic</td>
<td>$e^{-ar}$</td>
</tr>
<tr>
<td>Gravity</td>
<td>$1/r^3$</td>
</tr>
<tr>
<td>SRP</td>
<td>independent</td>
</tr>
<tr>
<td>Magnetic</td>
<td>$1/r^3$</td>
</tr>
</tbody>
</table>

- **Aerodynamic**: Strong dependence on solar activity and also day/night.
- **Gravity**: Almost no dependence on solar activity but care for eclipse!
SRP

SRP can be the primary source of disturbance torque. The lifetime of a GEO satellite is often controlled by the mass budget available for stationkeeping and attitude control fuel. Designers must avoid center-of-mass to center-of-pressure offsets.

Different disturbance torques must be considered.

The aerodynamic torque is dominant.
1.52-m diameter circular cylinder, 9.14 m in length

NASA SP-8058
The torque due to cosmic dust was evaluated for the cylinder normal to the stream and a separation distance of 0.36 m between the center of mass and center of pressure. Evaluation of the gravity gradient torque was carried out for an angle of 1° between the cylinder principal axis of inertia (the symmetry axis) and the Earth radius vector. The magnetic torque was computed assuming a 1-A current in a single loop of wire around the length of the cylinder oriented to give the maximum torque. The aerodynamic torque was computed using free-molecular flow theory, and the radiation torque was evaluated using a reflectivity of one.
How to stabilize the spacecraft with respect to disturbance torques?

1. **Passive attitude control**

\[
\frac{d(\vec{H})}{dt} = \frac{d(\vec{I} \, \vec{\omega})}{dt} = -\vec{T}_{\text{disturb}}
\]

**EXPLOIT!**
Gravity-Gradient Stabilization

What is the end effect of the gravity gradient?

\[
\begin{align*}
I_x \ddot{\phi}_x &= T_x + (I_y - I_z)\omega_y \omega_z \\
I_y \ddot{\omega}_y &= T_y + (I_z - I_x)\omega_x \omega_z \\
I_z \ddot{\omega}_z &= T_z + (I_x - I_y)\omega_x \omega_y \\
\end{align*}
\]

\[
\vec{T} = 3\omega_0^2 \begin{bmatrix} (I_z - I_y)\phi & (I_z - I_x)\theta & 0 \end{bmatrix}^T
\]

We deal with nonlinear equations

⇒ stable and unstable equilibrium positions

⇒ perform a stability analysis
Far Side of the Moon?

With the following reasoning, we will also answer why the moon keeps the same face turned towards the Earth at all times…

Near side of the moon

Far side of the moon
Notations

\[ X: \parallel \vec{v} \]

\[ Z: \parallel \vec{r} \quad \text{Local vertical} \]

Yaw \( \psi \)

Pitch \( \theta \)

Roll \( \phi \)
Governing Equations

\[ I_x \ddot{\omega}_x + (I_z - I_y) \omega_y \omega_z = 3 \omega_0^2 (I_z - I_y) \phi \]
\[ I_y \ddot{\omega}_y + (I_x - I_z) \omega_x \omega_z = 3 \omega_0^2 (I_z - I_x) \theta \]
\[ I_z \ddot{\omega}_z + (I_y - I_x) \omega_x \omega_y = 0 \]
\[ \omega_x = \dot{\phi} + \omega_0 \psi \]
\[ \omega_y = \dot{\theta} + \omega_0 \]
\[ \omega_z = \dot{\psi} - \omega_0 \phi \]

First-order terms (linearization)

\[ \ddot{\theta} - 3 \omega_0^2 \frac{I_z - I_x}{I_y} \theta = 0 \]
\[ \dot{\phi} + 4 k_\phi \omega_0^2 \phi + (1 - k_\phi) \omega_0 \psi = 0 \]
\[ \ddot{\psi} + k_\psi \omega_0^2 \psi - (1 - k_\psi) \omega_0 \phi = 0 \]

Pitch motion is decoupled from the other two motions

What can you tell about stability?
Stability

\[
\begin{align*}
\ddot{\theta} - 3\omega_0^2 \frac{I_z - I_x}{I_y} \theta &= 0 \\
\ddot{\phi} + 4k_\phi \omega_0^2 \phi + (1 - k_\phi) \omega_0 \psi &= 0 \\
\dot{\psi} + k_\psi \omega_0^2 \psi - (1 - k_\psi) \omega_0 \dot{\phi} &= 0
\end{align*}
\]

\[\theta = 0 \text{ is stable if } I_x > I_z\]

\[\psi = 0, \phi = 0 \iff \text{compute characteristic equation}\]

\[
\begin{vmatrix}
p^2 + 4\omega_0^2 k_\phi & p\omega_0 (1 - k_\phi) \\
-p\omega_0 (1 - k_\psi) & p^2 + \omega_0^2 k_\psi
\end{vmatrix} = p^4 + p^2 \omega_0^2 (1 + 3k_\phi + k_\phi k_\psi) + 4\omega_0^4 k_\phi k_\psi = 0
\]
Stability

\[ p^4 + p^2 \omega_0^2 (1 + 3k_\phi + k_\phi k_\psi) + 4\omega_0^4 k_\phi k_\psi = p^4 + bp^2 + c = 0 \]

Complex-conjugate solutions \( \Rightarrow \) some will have a positive real part \( \Rightarrow \) UNSTABLE

\[ b^2 < 4c \]

\[ p^2 = \frac{-b \pm \sqrt{b^2 - 4c}}{2} \]

\[ 1 + 3k_\phi + k_\phi k_\psi < 4\sqrt{k_\phi k_\psi} \]

\[ 1 + 3k_\phi + k_\phi k_\psi > 4\sqrt{k_\phi k_\psi} \]

\[ b^2 > 4c \]

Case 1: \( I_z < I_x < I_y \)
Case 2: \( I_y < I_z < I_x \)

Case 1: \( k_\phi k_\psi > 0 \)
Case 2: \( k_\phi < 0, k_\psi < 0 \)
Stability

\[ k_\varphi = \frac{B - A}{C} \]

Interesting zone (case 1)

\[ A = I_x, B = I_y, C = I_z \]
In Summary

The design rule is

\[ I_z < I_x < I_y \]

\[ \downarrow \quad \downarrow \quad \downarrow \]

Yaw  Roll  Pitch

What is the end effect of the gravity gradient?

A spacecraft tends to align its axis of minimum moment of inertia vertically.
Practical Consideration #1

The restoring torque depends upon the difference between principal moments.

\[
\ddot{\theta} - 3\omega_0^2 \frac{I_z - I_x}{I_y} \theta = 0
\]

\[
\ddot{\phi} + 4k_\phi \omega_0^2 \phi + (1 - k_\phi) \omega_0 \dot{\psi} = 0
\]

\[
\ddot{\psi} + k_\psi \omega_0^2 \psi - (1 - k_\psi) \omega_0 \dot{\phi} = 0
\]

GEOSAT: 800 km altitude (1985)
20-foot boom with 100-pound end

6-meter boom with 3-kg end
Practical Consideration #2

Undamped motion $\Rightarrow$ damping is needed to remove pendulum-like oscillations due to disturbances $\Rightarrow$ magnetic hysteresis rods or dampers

\[ \ddot{\theta} - 3\omega_0^2 \frac{I_z - I_x}{I_y} \theta = 0 \]

\[ \ddot{\phi} + 4k_\phi \omega_0^2 \phi + (1 - k_\phi) \omega_0 \dot{\psi} = 0 \]

\[ \ddot{\psi} + k_\psi \omega_0^2 \psi - (1 - k_\psi) \omega_0 \dot{\phi} = 0 \]

GEOSAT: Eddy current damper at its tip mass
Practical Consideration #3

The spacecraft is completely free to rotate about its vertical axis ⇒ add a momentum wheel with its axis perpendicular to the spacecraft vertical axis.
Far Side of the Moon

1. Initially, when the earth-moon system was young, the force of gravity from the earth produced bulges on the moon. The moon is therefore slightly elongated along the axis which points toward Earth.

2. Then, the moon aligned its axis of minimum moment of inertia vertically. The local vertical in the Moon-Earth system points toward the Earth.

3. This is why we do not see the far side of the moon: the spin of the moon is equal to its revolution around the earth (27.3 days).

Remark: This is true for a circular orbit. Due to the eccentricity of the moon’s orbit, we may actually observe about 59% of the moon's total surface.
The Moon is slightly elongated in shape, with its long axis perpetually pointing toward Earth. (The elongation is highly exaggerated in this diagram.)
Moon’s Strange Bulge Finally Explained

By Sara Goudarzi
Staff Writer
posted: 03 August 2006
02:10 pm ET

An eccentric orbit in the Moon's distant past might be responsible for the mysterious bulge around its middle, scientists say.

The excess material around the lunar equator has been known since 1799 when French mathematician Pierre-Simon Laplace first noticed it. The reason, however, has been a mystery until now.

The Moon's peculiar shape can be explained if the satellite moved in an eccentric oval-shaped orbit 100 million years after its violent formation, when the satellite hadn't yet solidified, the researchers say.

It was like a big ball of molasses and all around the equator it got deformed, study team member Ian Garrick-Bethell of the Massachusetts Institute of Technology told SPACE.com.

Around that time, conditions, such as orbit shape and position, were optimal for this "ball of molasses" to cool down and become the solid moon that we now know.

Today, the Moon's orbit around the Earth is nearly circular.

To predict the Moon's position and orbit millions of years ago, Garrick-Bethel and colleagues extrapolated backwards from ancient records of the timing of historical solar eclipses and of changes in the distance between the Earth and Moon.

http://www.space.com/scienceastronomy/060803_moon_shape.html
Spin Stabilization

The intrinsic gyroscopic stiffness of a spinning body is used to maintain its orientation in inertial space.

Unlike gravity-gradient stabilization, we maintain here the orientation in an inertial space (conservation of angular momentum).

Satellites intended for GEO are usually spin stabilized for the required orbit burns.
Spin Stabilization

No spin, constant disturbing torque, along one axis

Spin, constant disturbing torque perpendicular to the angular momentum

\[ T = I \ddot{\theta} \]

\[ \theta = \frac{1}{2} \frac{T}{I} t^2 \]

Quadratic growth

Linear growth inversely proportional to H, hence to the angular velocity

\[ \vec{T} = \frac{\Delta \vec{H}}{\Delta t}, \quad \theta = \frac{\Delta H}{H} = \frac{T \Delta t}{H} \]
Spin Stabilization

But remember Explorer 1!

A real (therefore flexible) body can spin stably only about the axis of maximum moment of inertia

⇒ the vehicle must be a “wheel” rather than a “pencil”

Meteosat was spin stabilized and possessed a nutation damper
Galileo was the first dual-spin planetary spacecraft: a spinning section rotates at about 3 rpm, and a "despun" section is counter-rotated to provide a fixed orientation for cameras and other remote sensors.
How to stabilize the spacecraft with respect to disturbance torques?

2. **Active attitude control** (determination + control)

\[
\frac{d(\vec{H})}{dt} = \frac{d(\vec{I} \, \vec{\omega})}{dt} = \vec{T}_{\text{disturb}} + \vec{T}_{\text{control}}
\]

APPLY!
Attitude Determination

- **ADCS**
  - Determination
    - Reference
      - Sun sensors
    - Horizon sensors
    - Magnetometers
    - Star trackers
  - Inertial
    - Gyro
  - Control
Analog Sun Sensors

A very basic implementation is to measure the output current of the solar cells.

The output current is proportional to the cosine of the angle of incidence of the solar radiation.
Digital Sun Sensors

The digital output of this sensor allows the plane in which the sun lies to be determined.

Two such detectors mounted orthogonally can determine the spacecraft-Sun vector in body axes.
Digital Sun Sensors

Filter ⇒
Sun Sensors: Voyager and ESEO

<table>
<thead>
<tr>
<th>Feature</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field of View</td>
<td>120° x 120°</td>
</tr>
<tr>
<td>Output</td>
<td>2-axis, 16 bit data word per axis, through CAN interface</td>
</tr>
<tr>
<td>Accuracy</td>
<td>0.02° (95% probability)</td>
</tr>
<tr>
<td>Noise Equivalent Angle</td>
<td>0.01°</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.01°</td>
</tr>
<tr>
<td>Data processing</td>
<td>In AOCS system</td>
</tr>
<tr>
<td>Operating temperature</td>
<td>-50°C to +80°C</td>
</tr>
<tr>
<td>Mass</td>
<td>250g</td>
</tr>
<tr>
<td>Power consumption</td>
<td>&lt;1W</td>
</tr>
<tr>
<td>Supply Voltage</td>
<td>5 Volts DC (±0.25 VDC, 135 mV ripple)</td>
</tr>
</tbody>
</table>

ESEO

Voyager
Horizon Sensors

Most common means of determining the Earth nadir vector

They sense the position of the horizon on each side of the spacecraft

They operate in the infrared (15µm CO₂ absorption); it also works during eclipse

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>±0.15° (3-sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oper. Range</td>
<td>4° around nadir</td>
</tr>
<tr>
<td>Spectral Band</td>
<td>14.0 to 16.0 microns</td>
</tr>
<tr>
<td>IFOV (horizon)</td>
<td>4.3° high by 6° wide</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>5 Volts DC (±0.25 VDC,</td>
</tr>
<tr>
<td>Power Draw</td>
<td>&lt; 1.70 Watt</td>
</tr>
<tr>
<td>Signal Out</td>
<td>CAN interface</td>
</tr>
<tr>
<td>Temperature</td>
<td>-20° C to +55°C</td>
</tr>
</tbody>
</table>
Magnetometers

They measure the direction and possibly the strength of the local magnetic field

Usually mutually orthogonal magnetometers

<table>
<thead>
<tr>
<th>Number of Axis</th>
<th>Three orthogonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>3-axis+1,16 bit data word per axis, through CAN interface</td>
</tr>
<tr>
<td>Orthogonality</td>
<td>0.5° (after calibration)</td>
</tr>
<tr>
<td>Frequency Response</td>
<td>200Hz</td>
</tr>
<tr>
<td>Field Measurement Range</td>
<td>600mG</td>
</tr>
<tr>
<td>Data processing</td>
<td>In AOCS system</td>
</tr>
<tr>
<td>Operating temperature</td>
<td>-40°C to +80°C</td>
</tr>
<tr>
<td>Mass</td>
<td>300g</td>
</tr>
<tr>
<td>Power consumption</td>
<td>&lt;1W</td>
</tr>
<tr>
<td>Supply Voltage</td>
<td>5 Volts DC (±0.25 VDC, 135 mV ripple)</td>
</tr>
</tbody>
</table>

4 magnetometers on ESEO spacecraft (3+1 for calibration)
Star Trackers

Optical device that measures the direction to one or more stars. There are 57 bright navigational stars in common use. One of the most used is Sirius (the brightest). For more complex missions entire starfield databases are used to identify orientation.

Most accurate source for a reference vector; absolute attitude determination accuracy down to the order of the arc-second.
Star Trackers: Voyager and ESEO

Detector: CCD matrix (288x384 pixels)
Field of view: 8.2x10.9°
Tracking capability: up to 10 stars
Dimension: 280x220x100mm
Mass: 3.6kg
Power consumption: <12W
Accuracy:
  - Star position: <4 arcsec
  - Attitude determination: <2 arcsec (roll<45 arcsec)
Tracking rate: up to 1°/sec
Update rate: 2 Hz
Gyroscopes

Devices that sense rotation in 3-space, without reliance on observation of external objects

A set of three orthogonal gyros measures the three components of the spacecraft’s inertial angular velocity

Gyros require initialization by some other means, as they can only measure *changes* in orientation. They also require inertial updates (drift)
How to stabilize the spacecraft with respect to disturbance torques?

2. **Active attitude control**
   
   (determination + control)

\[
\frac{d(\vec{H})}{dt} = \frac{d(\vec{I} \vec{\omega})}{dt} = \vec{T}_{\text{disturb}} + \vec{T}_{\text{control}}
\]

**APPLY!**
Active Control

ADCS

Determination

Reference
- Star sensors
- Horizon sensors
- Sun sensors
- Star trackers

Inertial
- Gyro

Passive
- Magnets
- Spin
- Gravity gradient

Control

Active
- Thrusters
- Mom. wheel
- Magneto-torquer
3-Axis Stabilized Spacecraft

Typical configuration of a 3-axis stabilized spacecraft

XMM-Newton
3-Axis Stabilized Spacecraft

- Torques
  - External
    - Thrusters
    - Magnetotorquers
  - Internal
    - Reaction wheels

Affect the total angular momentum

Necessary

Total angular momentum conserved but influence spacecraft orientation

Optional
Reaction Wheels

Spinning flywheels mounted on a central bearing whose rate of rotation can be adjusted by an electric motor.

Exchange momentum with the spacecraft by changing wheel speed but **no influence on the total angular momentum.**
Lunar Reconnaissance Orbiter
Video: Hardware in the loop simulator
Reaction Wheels: Usefulness

1. **Resist disturbing torques**: external torques give rise to unwanted angular momentum. The control system applies control torques to the reaction wheels to leave the spacecraft angular momentum unchanged.

Example: when a clockwise disturbance torque is imposed on the spacecraft, the attitude control system holds attitude constant by rotating a reaction wheel counterclockwise.
Reaction Wheels: Momentum Dumping

When disturbing torques do not average out over one orbit, constant wheel speed increase is necessary to hold the spacecraft.

There is a risk to “saturate” the wheel; the wheel is spinning too fast and cannot counterbalance further disturbing torques.

The stored momentum needs to be cancelled; this process is called momentum dumping.

Thrusters or magneto-torquers are used to hold the spacecraft stationary while reducing wheel speed.
2. **Slewing maneuvers**: rotate the spacecraft by very small amounts (e.g., keep a telescope pointed at a star) by slowing down or accelerating the wheel.

Example: from cruise attitude to target and return to cruise attitude: momentum is borrowed from the wheels and then returned to the wheel. There is no net change of momentum of the wheel in this case.
Wheels are not operated near 0 rpm, because of nonlinear wheel friction.

The rotational axis of a wheel is usually aligned with a vehicle control axis; the vehicle must carry one wheel per axis for full attitude control.

Redundancy is usually desired, requiring four or more wheels, in a position oblique to all axes.
## Reaction Wheels: Honeywell

<table>
<thead>
<tr>
<th>PERFORMANCE ITEM</th>
<th>UNIT</th>
<th>HR12</th>
<th>CAPABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>N-m-s</td>
<td>12, 25, 50(^\circ)</td>
<td>25, 50, 75</td>
</tr>
<tr>
<td>Reaction torque</td>
<td>N-m</td>
<td></td>
<td>0.1 to 0.2(^a)</td>
</tr>
<tr>
<td>Nominal</td>
<td>N-m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extended</td>
<td>N-m</td>
<td></td>
<td>up to 0.4 ((@3000) rpm)(^d)</td>
</tr>
<tr>
<td>Rotor Balance(^d)</td>
<td>g-cm</td>
<td>0.15, 0.24, 0.44</td>
<td>0.22, 0.35, 0.48</td>
</tr>
<tr>
<td>Static</td>
<td>g-cm(^d)</td>
<td>2.2, 4.6, 9.1</td>
<td>4.6, 9.1, 13.7</td>
</tr>
<tr>
<td>Dynamic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Power</td>
<td>Watts</td>
<td>105, 195(^a)</td>
<td></td>
</tr>
<tr>
<td>Steady State ((@6000) rpm)</td>
<td>Watts</td>
<td>&lt; 22 typical</td>
<td></td>
</tr>
<tr>
<td>Bus Voltage Range</td>
<td>Volts</td>
<td>14 up to 80</td>
<td></td>
</tr>
<tr>
<td>Wheel Speed</td>
<td>rpm</td>
<td>± 6000</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>kg</td>
<td>6.0, 7.0, 9.5</td>
<td>7.5, 8.5, 10.6</td>
</tr>
<tr>
<td>Integrated Wheel Outline (Height x Width)</td>
<td>mm</td>
<td>159 x 316</td>
<td>159 x 366</td>
</tr>
<tr>
<td>Separate Electronics Outline</td>
<td>mm</td>
<td>WU H148X316D</td>
<td>WU H148X366D</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WDE H60XW169XL230</td>
<td>WDE H60XW169XL230</td>
</tr>
<tr>
<td>Life</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Storage</td>
<td>Years</td>
<td>&gt; 5</td>
<td></td>
</tr>
<tr>
<td>On-orbit Operation</td>
<td>Years</td>
<td>&gt; 15</td>
<td></td>
</tr>
<tr>
<td>Radiation Hardness Capability</td>
<td>Krads (Si)</td>
<td>&gt; 300</td>
<td></td>
</tr>
<tr>
<td>Parts Screening</td>
<td></td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Operational Temperature Range (Qual)</td>
<td>°C</td>
<td>-30 to 70</td>
<td></td>
</tr>
<tr>
<td>Vibration</td>
<td>Grms</td>
<td>13.8</td>
<td></td>
</tr>
<tr>
<td>Interface</td>
<td>NA</td>
<td>Analog/Digital</td>
<td></td>
</tr>
</tbody>
</table>
THE THREE AXIS CONTROL OF THE HUBBLE SPACE TELESCOPE USING TWO REACTION WHEELS AND MAGNETIC TORQUER BARS FOR SCIENCE OBSERVATIONS

Sun Hur-Diaz, John Wirzburger, and Dan Smith

The Hubble Space Telescope (HST) is renowned for its superb pointing accuracy of less than 10 milli-arcseconds absolute pointing error. To accomplish this, the HST relies on its complement of four reaction wheel assemblies (RWAs) for attitude control and four magnetic torquer bars (MTBs) for momentum management. As with most satellites with reaction wheel control, the fourth RWA provides for fault tolerance to maintain three-axis pointing capability should a failure occur and a wheel is lost from operations. If
Similar in principle to reaction wheels, but fixed speed and mounted on gimbals that tilt the rotor’s angular momentum.

They provide 50 times more control torque than reaction wheels for the same mass and power.

Two applications:

*Large satellites (ISS)*
*Agile satellites (Pleiades)*
Control Moment Gyros

CMG (ISS)

Honeywell video
Control Moment Gyros: Pleiades

**CMG 15-45S overview**

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Momentum</strong></td>
<td>15 Nms</td>
</tr>
<tr>
<td><strong>Peak output torque</strong></td>
<td>45 Nm</td>
</tr>
<tr>
<td><strong>Range of motion</strong></td>
<td>Infinite</td>
</tr>
<tr>
<td><strong>CMG - Mechanism Mass</strong></td>
<td>15.7 kg</td>
</tr>
<tr>
<td><strong>Power consumption (incl. electronics)</strong></td>
<td>17 W (steady-state) 25 W (worst case)</td>
</tr>
<tr>
<td><strong>Stiffness</strong></td>
<td>&gt; 120 Hz</td>
</tr>
<tr>
<td><strong>CMG - Mechanism volume</strong></td>
<td>ø 270 mm, H 350 mm</td>
</tr>
<tr>
<td><strong>CMG - Mechanism footprint</strong></td>
<td>ø 200 mm</td>
</tr>
</tbody>
</table>
Magnetorquers

When a voltage is applied across a coil winding, a current is created, setting up a magnetic dipole. This dipole interacts with the Earth's magnetic field, causing the coil to attempt to align its own magnetic field in a direction opposite to that of the Earth's.

They may be used for dumping excess angular momentum from reaction wheels (e.g., Hubble)

Torque rod = helical coil
Thrusters

Common and effective means of providing spacecraft attitude control.

Common on satellites intended to operate in relatively high orbit, where a magnetic field will not be available for angular momentum dumping.

Potentially largest source of torque

Usually a redundant set of thrusters

Thrusters for orbit control may or may not be used as attitude thrusters as well
Thrusters

10N hydrazine thruster ($N_2H_4$)

Attitude thrusters?
Many control situations are satisfactorily achieved by using a simple PID (proportional, integral and differential).

The proportional value determines the reaction to the current error, the integral determines the reaction based on the sum of recent errors and the derivative determines the reaction to the rate at which the error has been changing.

**Very important:** the fundamental structural modes cannot fall near or within the desired ADCS bandwidth. If it is impossible, the structural modes must be modeled as part of the overall attitude control loop.
On-Board Computer (Controller)

Control Techniques
- Classical Control (frequency)
- Modern Control (time)
- Optimal, $H_{-\infty}$ (robust methods), Adaptive Control
SO MANY OPTIONS FOR DETERMINATION AND CONTROL!

WHY ???
Important Features

### Attitude determination
- Performance
- Constraints
- Availability
- Multiple sensors

### Attitude control
- Configuration
- Performance
- Constraints
- Momentum dumping
## Attitude Determination: Performance

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Accuracy, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun sensor</td>
<td>0.01-0.1 (angle that the sun subtends)</td>
</tr>
<tr>
<td>Horizon sensor</td>
<td>0.02-0.03 (Earth oblateness + fuzziness of horizon)</td>
</tr>
<tr>
<td>Magnetometer</td>
<td>1 (variability and uncertainty of the magnetic field)</td>
</tr>
<tr>
<td>Star tracker</td>
<td>0.001</td>
</tr>
<tr>
<td>Gyroscope</td>
<td>0.01/h (drift)</td>
</tr>
</tbody>
</table>
## Attitude Determination: Constraints

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Features</th>
<th>Weight (kg)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun sensor</td>
<td>Simple, cheap</td>
<td>[0.1 - 2]</td>
<td>[0 - 3]</td>
</tr>
<tr>
<td>Horizon sensor</td>
<td>Expensive</td>
<td>[1 - 3]</td>
<td>[1 - 10]</td>
</tr>
<tr>
<td>Magnetometer</td>
<td>Cheap</td>
<td>[0.3 - 1]</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Star tracker</td>
<td>Expensive, complex, slow</td>
<td>[2 - 5]</td>
<td>[5 - 20]</td>
</tr>
<tr>
<td>Gyroscope</td>
<td>Costly, rapid</td>
<td>[1 - 15]</td>
<td>[10 - 200]</td>
</tr>
</tbody>
</table>

For very rapid and accurate sensing, star trackers and gyroscopes are often used together.
## Attitude Determination: Availability

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun sensor</td>
<td>Eclipse (intermittent use)</td>
</tr>
<tr>
<td><strong>Horizon sensor</strong></td>
<td><strong>Continuous coverage</strong></td>
</tr>
<tr>
<td>Magnetometer</td>
<td>Continuous coverage, but below 6000 km</td>
</tr>
<tr>
<td>Star tracker</td>
<td>Intermittent use if the tracker does not see the reference stars (field of view)</td>
</tr>
<tr>
<td><strong>Gyroscope</strong></td>
<td><strong>Continuous coverage</strong></td>
</tr>
</tbody>
</table>
Attitude Determination: Multiple Sensors

How many directions should we sense?

**Full 3-axis knowledge requires at least two directions:** e.g., the sun vector and the orientation of the Earth’s magnetic field

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Direction</th>
<th>I/R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun sensor</td>
<td>One</td>
<td>Absolute</td>
</tr>
<tr>
<td>Horizon sensor</td>
<td>One</td>
<td>Absolute</td>
</tr>
<tr>
<td>Magnetometer</td>
<td>One</td>
<td>Absolute</td>
</tr>
<tr>
<td>Star tracker</td>
<td>One or two (number of stars tracked)</td>
<td>Absolute</td>
</tr>
<tr>
<td>Gyroscope</td>
<td>One</td>
<td>Relative</td>
</tr>
</tbody>
</table>

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## Attitude Control: Configuration

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control during kick-stage firing</td>
<td>Spin stabilization</td>
</tr>
<tr>
<td>Coarse control (&gt;10 deg)</td>
<td>Spin stabilization or gravity gradient</td>
</tr>
<tr>
<td>Low-accuracy (&gt;0.1 deg)</td>
<td>3-axis or dual spin</td>
</tr>
<tr>
<td>High-accuracy (&lt;0.1 deg)</td>
<td>3-axis</td>
</tr>
</tbody>
</table>
Attitude Control: Performance

Passive
- A few degrees
- Pointing in one direction

Active
+ 0.001 - 1 degrees
+ Pointing in several directions

<table>
<thead>
<tr>
<th>Component</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaction wheels</td>
<td>~0.005 deg</td>
</tr>
<tr>
<td>CMGs</td>
<td>0.1 deg</td>
</tr>
<tr>
<td>Thrusters</td>
<td>0.1 deg</td>
</tr>
<tr>
<td>Magnetorquers</td>
<td>1-2 deg</td>
</tr>
</tbody>
</table>
Attitude Control: Constraints

Passive

- Simple, no power
- Cheap

Active

- Complex, power required ([10 – 100 W] for wheels)
- Expensive
## Attitude Control: Availability

<table>
<thead>
<tr>
<th></th>
<th>Near Earth only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetotorquer</td>
<td>Used in any environment (but lifetime constraints !)</td>
</tr>
<tr>
<td>Thrusters</td>
<td>Wheels may be saturated due to secular disturbances and must be fitted with</td>
</tr>
<tr>
<td></td>
<td>magnetotorquers or thrusters (momentum dumping)</td>
</tr>
<tr>
<td>Reaction wheels</td>
<td></td>
</tr>
</tbody>
</table>
Typical Configurations

\(<0.1^\circ\)  3-axis stabilized
star sensors + gyros
reaction wheels and thrusters
flexible effects modeled in the controller
possibly vibration-isolated payload platform

\(0.1^\circ – 1^\circ\)  3-axis stabilized
star sensors + horizon sensors
reaction wheels and thrusters
flexible effects not modeled in the controller
Typical Configurations

1° – 5°  3-axis stabilized or spin stabilization
sun sensors + horizon sensors
reaction wheels or magnetorquers (thrusters for spin stabilization)

> 5°  gravity-gradient stabilization possible
**Example: Mars Express**

| ADCS | 8 attitude thrusters (10 N each) + star trackers + gyros + sun sensors + 4 reaction wheels (12 NMs) |

Typical configuration for accurate pointing
ADCS Failure Example: Hubble

Defective solar panels, recovery thanks to servicing mission

First natural frequency ?
Consequences for ADCS ?
ADCS Failure Example: Hubble

0.007”

Payload → 4.5 kW

Nonlinear!

New arrays (SM1)

Environment

2.6m x 7.1m
0.1 Hz, ξ<1%

Eclipse
(60°C/-85°C)

warped appearance
ADCS Design

Mission: orbit, cost, lifetime, payload, slewing maneuvers

Thermal control
Power (solar arrays)
Communications (antennas)
Propulsion

Sensor selection
Computational architecture
Actuation device selection
Configuration (3-axis/spinning)

Power (ADCS power consumption)
Propulsion (thruster type + amount of propellant)
Structure (center of mass + inertia constraints, flexibility, sensor and thruster locations)

Inputs
Outputs
Lecture 13

Attitude Determination and Control