Fundamental Aspects on Structural Dynamics of Spacecraft

Adriano Calvi, PhD

ESA / ESTEC, Noordwijk, The Netherlands

This presentation is distributed to the students of the University of Liege
(Satellite Engineering Course - Offered Spring 2008)
This presentation is not for further distribution
Foreword

• This 4-hours course on “Structural Dynamics of Spacecraft” intends to explain basic notions as well as some “advanced” concepts with minimum mathematics
• The content is the result of the author’s experience acquired through his involvement with research and industrial activities mainly at the European Space Agency and Alenia Spazio
• The course is specifically tailored for university students
• The focus is on: “What a spacecraft system engineer should know about structural dynamics and loads”
Fundamental Aspects on Structural Dynamics of Spacecraft (1)

• Introduction
  – Preliminary concepts and terminology
  – Requirements for spacecraft structures
  – The role of structural dynamics in a space project

• Dynamic analysis types
  – Real eigenvalue
  – Frequency response
  – Transient response
  – Shock response
  – Random vibration

• The effective mass concept
• Structural loading and preliminary design
Fundamental Aspects on Structural Dynamics of Spacecraft (2)

• Payload-launcher Coupled Loads Analysis (CLA)
• Mechanical tests
  – Acoustic noise test
  – Shock test
  – Random vibration test
  – Sinusoidal vibration test
  – Modal survey test
• Sine vibration testing and “notching”
• Mathematical model updating and validation
• Summary and conclusive remarks
  – Bibliography
Introduction – Preliminary concepts

Structural dynamics is the study of structures subjected to a mechanical environment which depends on time and leading to a movement

- Excitation transmission types (mechanical & acoustic)
- Type of time functions (sinusoidal, transient, random)
- Type of frequencies involved (low frequency, broadband)
- Domain of analysis (time domain, frequency domain)
- Structure representation with a mathematical model (continuous or discrete)
- Substructuring
- Optimization and model updating
**Preliminary concepts and terminology**

- The parameter most commonly used (in the industry) to “define the motion of a mechanical system” is the acceleration.
- Typical ranges of acceleration of concern in aerospace structures are from 0.01 g to 1000 g.
- Frequency (Hz or rad/s) and “octave”
- Vibroacoustics, pressure (N/m²) and Sound Pressure Level (dB)
- Random vibration and (acceleration) Power Spectral Density (G²/Hz)
- Shock Response Spectrum
- Root mean square (rms) = square root of the mean of the sum of all the squares
  - Note 1: the decibel is a tenth of a bel, the logarithm (base 10) of a power ratio (it is accepted that power is proportional to the square of the rms of acceleration, velocity, pressure, etc.)
  - Note 2: it must be emphasized that dB in acoustics is not an unit of acoustic pressure but simply a power ratio with respect to a reference pressure which must be stated or clearly implicit
Overview and other relevant terms

- Spacecraft, satellite, launcher, platform, bus, payloads…
- Quasi-static loads (events, tests…)
- Static and Dynamic loads (events, tests…)
- Natural frequency, natural mode, resonance
- Transient vibrations (events) & time histories
- Low, medium and high frequency range
- Fourier transform & frequency response
- Load factors and “CoG net accelerations”
- Qualification and acceptance tests
- Limit loads, design limit loads, yield & ultimate loads…
- Factors of safety, margins of safety, qualification & acceptance factors
Typical Requirements for Spacecraft Structures

• Strength
• Structural life
• Structural response
• Stiffness
• Damping
• Mass Properties
• Dynamic Envelope
• Positional Stability
• Mechanical Interface
Examples of (Mechanical) Requirements (1)

- The satellite shall be compatible with 2 launchers (potential candidates: VEGA, Soyuz in CSG, Rockot, Dnepr)...
- The satellite and all its units shall withstand applied loads due to the mechanical environments to which they are exposed during the service-life...
- Design Loads shall be derived by multiplication of the Limit Loads by a design factor equal to 1.25 (i.e. DL = 1.25 x LL)
- The structure shall withstand the worst design loads without failing or exhibiting permanent deformations.
- Buckling is not allowed.
- The natural frequencies of the structure shall be within adequate bandwidths to prevent dynamic coupling with major excitation frequencies...
- The spacecraft structure shall provide the mounting interface to the launch vehicle and comply with the launcher interface requirements.
Examples of (Mechanical) Requirements (2)

- All the Finite Element Models (FEM) prepared to support the mechanical verification activities at subsystem and satellite level shall be delivered in NASTRAN format.
- The FEM of the spacecraft in its launch configuration shall be detailed enough to ensure an appropriate derivation and verification of the design loads and of the modal response of the various structural elements of the satellite up to 140 Hz.
- A reduced FEM of the entire spacecraft correlated with the detailed FEM shall be delivered for the Launcher Coupled Loads Analysis (CLA)…
- The satellite FEMs shall be correlated against the results of modal survey tests carried out at complete spacecraft level, and at component level for units above 50 kg…
- The structural model of the satellite shall pass successfully qualification sine vibration Test.
- The flight satellite shall pass successfully acceptance sine vibration test.
The Role of Structural Dynamics in a Space Project

• Mechanical environment definition (structural response and loads identification by analysis and test)
  – Launcher/Payload coupled loads analysis
  – Random vibration and vibroacoustic analyses
  – Jitter analysis
  – Test predictions (e.g. sine test by frequency response analysis)
  – Test evaluations (sine, acoustic noise…)
  – …

• Structural identification (by analysis and test)
  – Modal analysis
  – Modal survey test and experimental modal analysis
  – Mathematical model updating and validation

• Design qualification and flight product acceptance
  – Qualification and Acceptance tests (sine, random, acoustic noise, shock)
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  – Transient response
  – Shock response
  – Random vibration

• The effective mass concept
• Structural loading and preliminary design
Dynamic analysis types

- Real eigenvalue analysis (undamped free vibrations)
  - Modal parameter identification, etc.
- Linear frequency response analysis (steady-state response of linear structures to loads that vary as a function of frequency)
  - Sine test prediction, transfer functions calculation, LV/SC CLA etc.
- Linear transient response analysis (response of linear structures to loads that vary as a function of time).
  - LV/SC CLA, base drive analysis, jitter analysis, etc.
- (Shock) response spectrum analysis
  - Equivalent sine level, test specifications for equipment etc.
- Vibro-acoustics (FEM/BEM, SEA) & Random vibration analysis
  - Vibro-acoustic test prediction
  - Loads analysis for base-driven random vibration
Reasons to compute normal modes (real eigenvalue analysis)

- To verify stiffness requirements
- To assess the dynamic interaction between a component and its supporting structure
- To guide experiments (e.g. modal survey test)
- To validate computational models (e.g. test/analysis correlation)
- As pre-requisite for subsequent dynamic analyses
- To evaluate design changes
- Mathematical model quality check (model verification)

\[
[M] \{ \ddot{u} \} + [K] \{ u \} = 0
\]
Real eigenvalue analysis

\[
[M]\{\dot{u}\} + [K][u] = 0
\]

\[
\{u\} = \{\phi\} \sin \omega t
\]

\[
([K] - \omega^2 [M])\{\phi\} = 0
\]

\[
\det ([K] - \omega^2 [M]) = 0
\]

\[
[K - \omega_i^2 M]\{\phi_i\} = 0 \quad i = 1, 2, 3 \ldots
\]

\[
f_i = \frac{\omega_i}{2\pi}
\]

\[
\{u\} = \sum_i (\phi_i)\xi_i
\]

\[
\{\phi_i\}^T[M]\{\phi_j\} = 0 \quad \text{if } i \neq j
\]

\[
\{\phi_j\}^T[M]\{\phi_j\} = m_j
\]

\[
\{\phi_j\}^T[K]\{\phi_j\} = 0 \quad \text{if } i \neq j
\]

\[
\{\phi_j\}^T[K]\{\phi_j\} = k_j
\]

- Note: mode shape normalization
- Scaling is arbitrary
- Convention: “Mass”, “Max” or “Point”
Mode shapes

- Cantilever beam
- Simply supported beam
Satellite Normal Modes Analysis

INTEGRAL Satellite (FEM size 120000 DOF’s)

Mode 1: 16.2 Hz

Mode 2: 18.3 Hz
Frequency Response Analysis

• Used to compute structural response to steady-state harmonic excitation
• The excitation is explicitly defined in the frequency domain
• Forces can be in the form of applied forces and/or enforced motions
• Two different numerical methods: direct and modal
• Damped forced vibration equation of motion with harmonic excitation:

\[
[M]\{x(t)\} + [B]\{x(t)\} + [K]\{x(t)\} = \{P(\omega)\}e^{i\omega t}
\]
Frequency response considerations

- If the maximum excitation frequency is much less than the lowest resonant frequency of the system, a static analysis is probably sufficient.
- Undamped or very lightly damped structures exhibit large dynamic responses for excitation frequencies near resonant frequencies.
- Use a fine enough frequency step size ($\Delta f$) to adequately predict peak response.
- Smaller frequency spacing should be used in regions near resonant frequencies, and larger frequency step sizes should be used in regions away from resonant frequencies.
Harmonic forced response with damping

Amplification Factor

Phase Lead $\theta$ (Degrees)

Forcing Frequency $\omega$

$\omega_n$

270°

360°

180°

1

2ζ
Transient Response Analysis

• Purpose is to compute the behaviour of a structure subjected to time-varying excitation
• The transient excitation is explicitly defined in the time domain
• Forces can be in the form of applied forces and/or enforced motions
• The important results obtained from a transient analysis are typically displacements, velocities, and accelerations of grid points, and forces and stresses in elements
• Two different numerical methods: direct and modal
• Dynamic equation of motion:

\[
[M]\{\ddot{u}(t)\} + [B]\{\dot{u}(t)\} + [K]\{u(t)\} = \{P(t)\}
\]
Modal Transient Response Analysis

\[
[M]\{u(t)\} + [K]\{u(t)\} = \{P(t)\}
\]

\[
\{u(t)\} = [\phi]\{\xi(t)\}
\]

\[
[M][\phi]\{\xi(t)\} + [K][\phi]\{\xi(t)\} = \{P(t)\}
\]

\[
\]

\[
m_i \ddot{\xi}_i(t) + k_i \dot{\xi}_i(t) = p_i(t)
\]

\[
\{u(t)\} = [\phi]\{\xi(t)\}
\]
Transient response consideration

• The integration time step must be small enough to represent accurately the variation in the loading

• The integration time step must also be small enough to represent the maximum frequency of interest (“cut-off frequency”)

• The cost of integration is directly proportional to the number of time steps

• Very sharp spikes in a loading function induce a high-frequency transient response. If the high-frequency transient response is of primary importance in an analysis, a very small integration time step must be used

• The loading function must accurately describe the spatial and temporal distribution of the dynamic load
Shock response spectrum (and analysis)

• Response spectrum analysis is an approximate method of computing the peak response of a transient excitation applied to a structure or component

• used in aerospace engineering to predict the peak response of equipment in a spacecraft that is subjected to an impulsive load due e.g. to stage separation

• There are two parts to response spectrum analysis: (1) generation of the spectrum and (2) use of the spectrum for dynamic response such as stress analysis

• Note: response spectrum analysis is not (normally) used in structural dynamics of spacecraft since the accuracy of the method may be questionable
Generation of a response spectrum (1)

(a) Transient Excitation

(b) Base Structure

(c) Transient Response

(d) Series of Oscillators

(f_1, f_2, f_3, \ldots, f_{\text{max}})

(e) Resonant Frequency

(f_1, f_2, f_3, \ldots, f_{\text{max}})

Peak Response
Generation of a response spectrum (2)

- the peak response for one oscillator does not necessarily occur at the same time as the peak response for another oscillator
- there is no phase information since only the magnitude of peak response is computed
- It is assumed in this process that each oscillator mass is very small relative to the base structural mass so that the oscillator does not influence the dynamic behaviour of the base structure
Shock Response Spectrum

(A) is the shock spectrum of a terminal peak sawtooth (B) of 500 G peak amplitude and 0.4 millisecond duration.
Random vibration (analysis)

- Random vibration is vibration that can be described only in a statistical sense
- The instantaneous magnitude is not known at any given time; rather, the magnitude is expressed in terms of its statistical properties (such as mean value, standard deviation, and probability of exceeding a certain value)
- Examples of random vibration include earthquake ground motion, wind pressure fluctuations on aircraft, and acoustic excitation due to rocket and jet engine noise
- These random excitations are usually described in terms of a power spectral density (PSD) function
- Note: in structural dynamics of spacecraft, the random vibration analysis is often performed with simplified techniques (e.g. based on “Miles’ equation”)
Random noise with normal amplitude distribution
Power Spectral Density (conceptual model)
Sound Pressure Level (conceptual model)
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Modal effective mass (1)

- It may be defined as the mass terms in a modal expansion of the drive point apparent mass of a kinematically supported system.
- This concept applies to structure with base excitation (but it has a more general applications...)
- It provides an estimate of the participation of a vibration mode, in terms of the load it will cause in the structure, when excited.

- Note: avoid using: “it is the mass which participates to the mode”!

\[
\{F_{\text{base}, k}\} = [M_{\text{eff}, k}] \left\{ 1 + \left( \frac{\omega}{\omega_k} \right)^2 H_k \left( \frac{\omega}{\omega_k} \right) \right\} \{\ddot{X}_j\}
\]
Modal effective mass (2)

\[
\tilde{M}_i = \frac{L_i^T L_i}{m_i} \quad L_i = \phi_i^T M \Phi_i \quad \tilde{M}_i = \frac{1}{m_i n_i^2} RR^T
\]

- The effective mass matrix can be calculated either by the “modal participation factors” or by using the modal interface forces.
- Normally only the values on the leading diagonal of the modal effective mass matrix are considered and expressed in percentage of the structure rigid body properties (total mass and second moments of inertia).
- The effective mass characterises the mode and it is independent from the eigenvector normalisation.
Modal effective mass (3)

- For the complete set of modes the summation of the modal effective mass is equal to the rigid body mass
- Contributions of each individual mode to the total effective mass can be used as a criterion to classify the modes (global or local) and an indicator of the importance of that mode, i.e. an indication of the magnitude of participation in the loads analysis
- It can be used to construct a list of important modes for the test/analysis correlation and it is a significant correlation parameter
- It can be used to create simplified mathematical models
## Example of Effective Mass table (MPLM test and FE model)

### Fundamental Aspects on Structural Dynamics of Spacecraft - A. Calvi

### Table: Effective Masses

<table>
<thead>
<tr>
<th>Mode</th>
<th>EFFECTIVE MASSES (Eigenvectors and Red. Mass Matrix)</th>
<th>Mode</th>
<th>EFFECTIVE MASSES (Eigenvectors and Full Mass Matrix)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Freq [Hz]</td>
<td>X [%]</td>
<td>Y [%]</td>
</tr>
<tr>
<td>1</td>
<td>13.22</td>
<td>0.0</td>
<td>88.5</td>
</tr>
<tr>
<td>2</td>
<td>13.82</td>
<td>90.2</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>16.92</td>
<td>0.9</td>
<td>6.0</td>
</tr>
<tr>
<td>4</td>
<td>19.44</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>20.60</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>21.79</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>21.92</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>8</td>
<td>21.55</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>9</td>
<td>22.75</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>24.17</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>24.60</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12</td>
<td>25.75</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>13</td>
<td>25.87</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>14</td>
<td>29.21</td>
<td>0.4</td>
<td>0.0</td>
</tr>
<tr>
<td>15</td>
<td>30.80</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>16</td>
<td>30.59</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>17</td>
<td>31.27</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>18</td>
<td>33.95</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>19</td>
<td>36.12</td>
<td>8.5</td>
<td>0.0</td>
</tr>
<tr>
<td>20</td>
<td>38.04</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>21</td>
<td>39.31</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>22</td>
<td>39.74</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>23</td>
<td>42.10</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>24</td>
<td>47.53</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>25</td>
<td>48.94</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>26</td>
<td>49.65</td>
<td>0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### Note:
- IDN: C = Ceiling, F = Floor, P = Portside, S = Starboardside
- ph. = in phase, o.p.h. = in opposition of phase
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A5 Typical Sequence of events

- Main cryogenic stage engine shutdown (H2) and separation
- Upper stage ignition
- Upper stage shutdown (H3)
- Fairing jettisoning (FJ)
- SRB flame-out (H1) and separation
- Main cryogenic stage engine ignition (H0)
- SRB ignition and lift-off
A5 Typical Longitudinal Static Acceleration
Sources of Structural Loadings (Launch)

Axial-Acceleration Profile for the Rockot Launch Vehicle
Axial Acceleration at Launcher/Satellite Interface (Engines Cut-off)
## Load Factors for Preliminary Design (Ariane 5)

<table>
<thead>
<tr>
<th>Critical flight events</th>
<th>Longitudinal</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static</td>
<td>Dynamic</td>
</tr>
<tr>
<td>Lift-off</td>
<td>- 1.7</td>
<td>± 1.5</td>
</tr>
<tr>
<td>Maximum dynamic pressure</td>
<td>- 2.7</td>
<td>± 0.5</td>
</tr>
<tr>
<td>SRB end of flight</td>
<td>- 4.55</td>
<td>± 1.45</td>
</tr>
<tr>
<td>Main core thrust tail-off</td>
<td>- 0.2</td>
<td>± 1.4</td>
</tr>
<tr>
<td>Max. tension case: SRB jettisoning</td>
<td></td>
<td>+ 2.5**</td>
</tr>
</tbody>
</table>
**Loads and Factors**
ECSS E-32-10

<table>
<thead>
<tr>
<th>Load Level</th>
<th>Satellite Test Logic</th>
<th>Common Design Logic</th>
<th>Expendable launch vehicles, pressurized hardware and manned system Test Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limit Loads - LL</td>
<td>x KQ</td>
<td>x KA</td>
<td>x Coef. A</td>
</tr>
<tr>
<td>Design Limit Loads DLL</td>
<td>x Coef. B</td>
<td>x Coef. C</td>
<td></td>
</tr>
<tr>
<td>x KQ</td>
<td>x KA</td>
<td>x KQ</td>
<td>x KA</td>
</tr>
<tr>
<td>QL</td>
<td>AL</td>
<td>QL</td>
<td>AL</td>
</tr>
<tr>
<td>DYL</td>
<td>DUL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Satellite</th>
<th>Launch vehicles and pressurised hardware</th>
<th>Man-rated systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$KQ \times K_x \times K_M$</td>
<td>$K_2 \times K_M$</td>
<td>$K_2 \times K_M$</td>
</tr>
<tr>
<td>B</td>
<td>$FOSY \times K_L$</td>
<td>$FOSY \times K_{MP} \times K_L$</td>
<td>$FOSY \times K_L$</td>
</tr>
<tr>
<td>C</td>
<td>$FOSU \times K_L$</td>
<td>$FOSU \times K_{MP} \times K_L$</td>
<td>$FOSU \times K_L$</td>
</tr>
</tbody>
</table>
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Launcher / Satellite C.L.A.

- **CLA**: simulation of the structural response to low frequency mechanical environment

- **Main Objective**: to calculate the loads on the satellite caused by the launch transients (lift-off, transonic, aerodynamic gust, separation of SRBs…)

- **Loads** (in this context): set of internal forces, displacements and accelerations that characterise structural response to the applied forces

- **Effects included in the forcing functions**: thrust built-up, engine shut-down/burnout, gravity, aerodynamic loads (gust), separation of boosters, etc.
Ariane-5 Dynamic Mathematical Model

- Dynamic effects up to about 100 Hz
- 3D FE models of EPC, EAP, UC
- Dynamic Reduction using Craig-Bampton formulation
- Incompressible or compressible fluids models for liquid propellants
- Structure/fluid interaction
- Nearly incompressible SRB solid propellant modeling
- Pressure and stress effects on launcher stiffness
- SRB propellant and DIAS structural damping
- Non-linear launch table effects
Sizing flight events (CLA with VEGA Launcher)

1. Lift-off (P80 Ignition and Blastwave)
2. Mach1/QMAX Gust
3. P80 Pressure Oscillations
4. Z23 Ignition
5. Z23 Pressure Oscillations
6. Z9 Ignition
CLA Output

- LV-SC interface accelerations
  - Equivalent sine spectrum

- LV-SC interface forces
  - Equivalent accelerations at CoG

- Internal responses
  - Accelerations,
  - Displacements
  - Forces
  - Stresses
  - ...
Payload / STS CLA

Lift-off Force Resultant in X [lbf]

Lift-off
Main Fitting
I/F Force
X Dir. [N]

Lift-off
Main Fitting
I/F Force
Z Dir. [N]

Lift-off
Keel Fitting
I/F Force
Y Dir. [N]
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Testing techniques – Introduction (1)

• Without testing, an analysis can give completely incorrect results
• Without the analysis, the tests can represent only a very limited reality

• Two types of tests according to the objectives to be reached:
  – Simulation tests for structure qualification or acceptance
  – Identification tests (a.k.a. analysis-validation tests) for structure identification (the objective is to determine the dynamic characteristics of the tested structure in order to “update” the mathematical model)

• Note: identification and simulation tests are generally completely dissociated. In certain cases (e.g. spacecraft sine test) it is technically possible to perform them using the same test facility)
Testing techniques – Introduction (2)

• Generation of mechanical environment
  – Small shakers (with flexible rod; electrodynamic)
  – Large shakers (generally used to impose motion at the base)
    • Electrodynamic shaker
    • Hydraulic jack shaker
  – Shock machines (pyrotechnic generators and impact machines)
  – Noise generators + reverberant acoustic chamber (homogeneous and diffuse field)

• Measurements
  – Force sensors, calibrated strain gauges
  – Accelerometers, velocity or displacement sensors
Classes of tests used to verify requirements (purposes)

- Development test
  - Demonstrate design concepts and acquire necessary information for design
- Qualification test
  - Show a design is adequate by testing a single article
- Acceptance test
  - Show a product is adequate (test each flight article)
- Analysis validation test
  - Provide data which enable to confirm critical analyses or to change (“update/validate”) our math models and redo analyses
Tests for verifying mechanical requirements (purposes)

• Acoustic test
  – Verify strength and structural life by introducing random vibration through acoustic pressure (vibrating air molecules)
  – *Note: acoustic tests at spacecraft level are used to verify adequacy of electrical connections and validate the random vibration environments used to qualify components*

• (Pyrotechnic) shock test
  – Verify resistance to high-frequency shock waves caused by separation explosives (*introduction of high-energy vibration up to 10,000 Hz*)
  – *System-level tests are used to verify levels used for component testing*

• Random vibration test
  – Verify strength and structural life by introducing random vibration through the mechanical interface (*typically up to 2000 Hz*)
Tests for verifying mechanical requirements (purposes)

• Sinusoidal vibration test
  – Verify strength for structures that would not be adequately tested in random vibration or acoustic testing
  – Note 1: cyclic loads at varying frequencies are applied to excite the structure modes of vibration
  – Note 2: sinusoidal vibration testing at low levels are performed to verify natural frequencies
  – Note 3: the acquired data can be used for further processing (e.g. experimental modal analysis)
  – Note 4: this may seem like an environmental test, but it is not. Responses are monitored and input forces are reduced as necessary (“notching”) to make sure the target responses or member loads are not exceeded.
Identification tests (modal survey test)

- The normal modes are the most appropriate dynamic characteristics for the identification of the structure
- Modal parameters (natural frequencies, mode shapes, damping, effective masses...) can be determined in two ways:

  – by a method with appropriation of modes, sometimes called phase resonance, which consists of successively isolating each mode by an appropriate excitation and measuring its parameters directly
  – by a method without appropriation of modes, sometimes called phase separation, which consists of exciting a group of modes whose parameters are then determined by processing the measurements
Marketed by: Eurockot  
Actually flight qualified

Manufactured by: Khrunichev

Launch site: Plesetsk

Capability: 950 kg @ 500 km

**Environment** | **Level**
--- | ---
**Sine vibration** | **Longitudinal**
1 g on [5-10] Hz  
1.5 g at 20 Hz  
1 g on [40-100] Hz  
**Lateral** = 0.625 g on [5-100] Hz

**Acoustic**
31.5 Hz = 130.5 dB  
63 Hz = 133.5 dB  
125 Hz = 135.5 dB  
250 Hz = 135.7 dB  
500 Hz = 130.8 dB  
1000 Hz = 126.4 dB  
2000 Hz = 120.3 dB

**Shock**
100 Hz = 50 g  
700 Hz = 800 g  
1000 Hz → 1500 Hz = 2000 g  
4000 Hz → 5000 Hz = 4000 g  
10000 Hz = 2000 g
Ariane 5 - Sine excitation at spacecraft base (sine-equivalent dynamics)
Ariane 5 – Acoustic noise spectrum under the fairing
Artemis and ATV in the LEAF
Shock machine
Random vibration test with slide table
Random vibration test: data-processing bandwidth

- The figures show how the data-processing bandwidth can affect a calculated power spectral density. Whether a PSD satisfies criteria for level and tolerance depends on the frequency bandwidth used to process the measured acceleration time history.
Fundamental Aspects on Structural Dynamics of Spacecraft (2)

• Payload-launcher Coupled Loads Analysis (CLA)
• Mechanical tests
  – Acoustic noise test
  – Shock test
  – Random vibration test
  – Sinusoidal vibration test
  – Modal survey test
• Sine vibration testing and “notching”
• Mathematical model updating and validation
• Summary and conclusive remarks
  – Bibliography
Herschel on Hydra
GOCE on ESTEC Large Slip Table

Herschel on ESTEC Large Slip Table
Sine vibration test

• The qualification of the satellite to low frequency transient is normally achieved by a base-shake test

• The input spectrum specifies the acceleration input that should excite the satellite, for each axis

• This input is definitively different from the mission loads, which are transient
The overtesting problem (causes)

• Difference in boundary conditions between test and flight configurations
  – during a vibration test, the structure is excited with a specified input acceleration that is the envelope of the flight interface acceleration, despite the amplitude at certain frequencies drops in the flight configuration (there is a feedback from the launcher to the spacecraft in the main modes of the spacecraft)

• The excitation during the flight is not a steady-state sine function and neither a sine sweep but a transient excitation with some cycles in a few significant resonance frequencies.

• The objective of notching of the specified input levels is to take into account the real dynamic response for the different flight events
Shock Response Spectrum and Equivalent Sine Input

- A shock response spectrum is a plot of maximum “response” (e.g. displacement, stress, acceleration) of single degree-of-freedom (SDOF) systems to a given input versus some system parameter, generally the undamped natural frequency.

\[
SRS(\omega) = \max_{\omega} |\ddot{x}(t, \omega)|
\]

\[
\ddot{x}(t, \omega) = \ddot{z}(t, \omega) + \ddot{u}(t)
\]

\[
\ddot{z}(t) + 2\zeta\omega_n \dot{z}(t) + \omega_n^2 z(t) = -\ddot{u}(t)
\]
SRS/ESI of the following transient acceleration:

\[ ESI = \frac{SRS}{Q} \]

\[ f_n = 10 \text{ Hz} \]

\[ f = 10 \text{ Hz} \]
ESI for Spacecraft

CLA (Coupled Load Analysis)

Difference is negligible for small damping ratios

\[ ESI = \frac{SRS}{Q} \]

\[ ESI = \frac{SRS}{\sqrt{Q^2 + 1}} \]
2DOF

Transient response

Frequency response at ESI level

SDOF

Transient response

Frequency response at ESI level
The effects of the sine sweep rate on the structural response

- The acceleration enforced by the shaker is a swept frequency function
- The sweep is amplitude modulated
- Acceleration transient response can be significantly lower than the steady-state frequency response

![Graph showing acceleration transient response at different sweep rates](image-url)
X RUN - INPUT APPLIED AT QUALIFICATION

ESI (equivalent sine)

Notching

Fundamental Aspects on Structural Dynamics of Spacecraft - A. Calvi
Y RUN - INPUT APPLIED AT QUALIFICATION

- WING Z+
- WING Z-
- GRADIOMETER CORE
- BENDING MOMENT

Freq. [Hz]

Acceleration [g]

Y INPUT

CLA Y INPUT Q=20

CLA Y INPUT Q=50

Fundamental Aspects on Structural Dynamics of Spacecraft - A. Calvi
MOMENT MEASURED BY FMD - Y QUALIFICATION RUN

Moment [Nm]

Freq. [Hz]
“(Sine) quasi static load test” (sine burst)
Fundamental Aspects on Structural Dynamics of Spacecraft (2)

• Payload-launcher Coupled Loads Analysis (CLA)
• Mechanical tests
  – Acoustic noise test
  – Shock test
  – Random vibration test
  – Sinusoidal vibration test
  – Modal survey test
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• Summary and conclusive remarks
  – Bibliography
Validation of Finite Element Models
(with emphasis on Structural Dynamics)

“Everyone believes the test data except for the experimentalist, and no one believes the finite element model except for the analyst”

“All models are wrong, but some are still useful”
Verification and Validation Definitions
(ASME Standards Committee: “V & V in Computational Solid Mechanics”)

• **Verification** (of codes, calculations): Process of determining that a model implementation accurately represents the developer’s conceptual description of the model and the solution to the model
  – Math issue: “Solving the equations right”

• **Validation**: Process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model
  – Physics issue: “Solving the right equations”

**Note**: objective of the validation is to maximise confidence in the predictive capability of the model
Terminology: Correlation, Updating and Validation

- **Correlation:**
  - the process of quantifying the degree of similarity and dissimilarity between two models (e.g. FE analysis vs. test)

- **Error Localization:**
  - the process of determining which areas of the model need to be modified

- **Updating:**
  - mathematical model improvement using data obtained from an associated experimental model (it can be “consistent” or “inconsistent”)

- **Valid model:**
  - model which predicts the required dynamic behaviour of the subject structure with an acceptable degree of accuracy, or “correctness”
Targets of the correlation
(characteristics that most affect the structure response to applied forces)

• Natural frequencies
• Mode shapes
• Modal effective masses
• Modal damping (note: normally there is no prediction of damping)
• ...
• Total mass, mass distribution, Centre of Gravity, inertia
• Static stiffness
• Interface forces
• …
Cross Orthogonality Check and Modal Assurance Criterion (MAC)

- The cross-orthogonality between the analysis and test mode shapes with respect to the mass matrix is given by:

\[
C = \Phi_m^T M \Phi_a
\]

- The MAC between a measured mode and an analytical mode is defined as:

\[
MAC_{rs} = \frac{\left(\phi_{mr}^T \phi_{as}\right)^2}{\phi_{mr}^T \phi_{mr} \phi_{as}^T \phi_{as}}
\]
### Columbus: Cross-Orthogonality Check up to 35 Hz (target modes)

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MPLM Modal Correlation

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<td>49.55</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Total [%]: 101.7, 91.4, 72.5, 62.1, 70.6, 104.9
Ref. Val: 6341, 6304, 6295, 20610, 23300, 24830

**Fundamental Aspects on Structural Dynamics of Spacecraft - A. Calvi**

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## Soho SVM – Cross-Orthogonality Check

| F.E.M. | TEST | Err. % | Freq. Hz | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| 1      | 2.87 | 35.86  |          | 0.87 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 2      | 0.00 | 37.24  |          | 0.47 | 0.87 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 3      | 4.17 | 45.99  |          |   |   | 0.87 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 4      | 4.78 | 47.46  |          |   |   |   | 0.77 | 0.24 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 5      | -3.39| 49.82  |          |   |   |   |   |   | 0.76 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 6      | 0.96 | 53.19  |          |   |   |   |   |   |   | 0.75 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 7      | 2.10 | 56.65  |          |   |   |   |   |   |   |   | 0.79 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 8      | 58.67|        |          |   |   |   |   | -0.28 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 9      | 60.24|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 10     | 3.30 | 64.30  |          |   |   |   |   |   |   |   |   |   |   |   | 0.61 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 11     | 66.40|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   | 0.32 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 12     | 67.50|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0.45 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 13     | 68.73|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | -0.43 |   |   |   |   |   |   |   |   |   |   |   |   |
| 14     | 69.68|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | -0.38 |   |   |   |   |   |   |   |   |   |   |   |
| 15     | 71.69|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0.37 | -0.33 |   |   |   |   |   |   |   |
| 16     | 72.71|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0.21 |   |   |   |   |   |   |   |
| 17     | 3.30 | 73.34  |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0.85 |   |   |   |   |   |
| 18     | 74.78|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 19     | 75.63|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 20     | 78.77|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 21     | 82.12|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 22     | 7.72 | 84.51  |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 23     | 5.54 | 86.31  |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 24     | 7.21 | 88.64  |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 25     | 5.45 | 89.29  |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 26     | 94.44|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 27     | 97.15|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 28     | 99.56|        |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

Note: The table represents cross-orthogonality check results for specific frequencies and test cases. The values indicate correlation coefficients between different frequency bands and test results. Higher values indicate better orthogonality.
### GOCE - MAC and Effective Mass

#### FINAL MAC BETWEEN TEST AND ANALYSIS

<table>
<thead>
<tr>
<th>TEST</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>9</th>
<th>14</th>
<th>18</th>
<th>22</th>
<th>29</th>
<th>30</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15.07</td>
<td>16.13</td>
<td>28.84</td>
<td>34.86</td>
<td>47.27</td>
<td>49.59</td>
<td>64.15</td>
<td>73.40</td>
<td>77.15</td>
<td>84.51</td>
<td>85.30</td>
<td>102.13</td>
</tr>
<tr>
<td></td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.88</td>
<td>0.98</td>
<td>0.96</td>
<td>0.06</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.06</td>
<td>0.87</td>
</tr>
</tbody>
</table>

**EFFECTIVE MASSES X DIRECTION**

**EFFECTIVE MASSES Z DIRECTION**
Aeolus STM: comparison of transfer functions

Sine test response, FEM predicted response and post-test (updated FEM) response
Lack of Matching between F.E. Model and Test

- Modelling uncertainties and errors (model is not completely physically representative)
  - Approximation of boundary conditions
  - Inadequate modelling of joints and couplings
  - Lack or inappropriate damping representation
  - The linear assumption of the model versus test non-linearities
  - Mistakes (input errors, oversights, etc.)

- Scatter in manufacturing
  - Uncertainties in physical properties (geometry, tolerances, material properties)

- Uncertainties and errors in testing
  - Measured data or parameters contain levels of errors
  - Uncertainties in the test set-up, input loads, boundary conditions etc.
  - Mistakes (oversights, cabling errors, etc.)
### Test-Analysis Correlation Criteria
The degree of similarity or dissimilarity establishing that the correlation between measured and predicted values is acceptable

<table>
<thead>
<tr>
<th>Item</th>
<th>Quality criterion ( ^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental bending modes of a spacecraft</td>
<td>MAC:</td>
</tr>
<tr>
<td></td>
<td>Eigenfrequency deviation:</td>
</tr>
<tr>
<td></td>
<td>&lt; 3 %</td>
</tr>
<tr>
<td>Modes with effective masses &gt; 10% of the total mass</td>
<td>MAC:</td>
</tr>
<tr>
<td></td>
<td>Eigenfrequency deviation:</td>
</tr>
<tr>
<td></td>
<td>&gt; 0.85</td>
</tr>
<tr>
<td>For other modes in the relevant frequency range (^b)</td>
<td>MAC:</td>
</tr>
<tr>
<td></td>
<td>Eigenfrequency deviation:</td>
</tr>
<tr>
<td></td>
<td>&lt; 10 %</td>
</tr>
<tr>
<td>Cross-orthogonality check</td>
<td>Diagonal terms:</td>
</tr>
<tr>
<td></td>
<td>Off-diagonal terms:</td>
</tr>
<tr>
<td></td>
<td>&gt; 0.90</td>
</tr>
<tr>
<td></td>
<td>&lt; 0.10</td>
</tr>
<tr>
<td>Damping</td>
<td>To take measured values as input for the response analysis.</td>
</tr>
<tr>
<td></td>
<td>To use realistic test inputs for this purpose.</td>
</tr>
<tr>
<td>Interface force and moment measurements</td>
<td>For modes with effective masses &gt; 10%: deviations of interface forces and moments &lt; 10%.</td>
</tr>
</tbody>
</table>

\( ^a \) The quality criteria given are not normative and are given as examples for achieving a satisfactory test-analysis correlation.

\( ^b \) The relevant frequency range is, in general, determined by the launcher excitation spectrum up to 100 Hz. This frequency range can, however, be extended due to, for example, high frequency launcher dynamic excitations or specific requirements for AOCS control purposes.

### ECSS-E-30-11A Proposed Test / Analysis Correlation Criteria
Model updating: the “objective functions”

Natural Frequency: \( d_f = \frac{f_a - f_e}{f_e} \)

Mode Shape: \( d_{\Phi} = \frac{\alpha \Phi_a - \Phi_e}{\Phi_e} \)  \( (\alpha: \text{modal scale factor}) \)

Effective Transmissibilities: \( d_{\tilde{T}} = \frac{\tilde{T}_a - \tilde{T}_e}{\tilde{T}_e} \)

Effective Masses: \( d_{\tilde{M}} = \frac{\tilde{M}_a - \tilde{M}_e}{\tilde{M}_e} \)

Objective Function

\[ F = b^T W b \]  \text{with}  \[ b = \begin{bmatrix} d_f \\ d_{\Phi} \\ d_{\tilde{T}} \\ d_{\tilde{M}} \end{bmatrix} \text{and} \ W = \begin{bmatrix} w_f \\ w_{\Phi} \\ w_{\tilde{T}} \\ w_{\tilde{M}} \end{bmatrix} \]
Summary and Conclusive Remarks

• The role of structural dynamics in a space project
• Dynamic analysis types
• The effective mass concept
• Structural loading and preliminary design
• Payload-launcher Coupled Loads Analysis
• Mechanical tests
• Sine vibration testing and “notching”
• Mathematical model updating and validation
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• ECSS-E-32-03 Structural finite element models
• ECSS-E-32-10 Structural factors of safety for spaceflight hardware
• ECSS-E-32-02 Structural design of pressurized hardware
• ECSS-E-30-10 Modal survey assessment
• ECSS-E-32-01 Fracture control

• ECSS-E-10-02 Space Engineering - Verification
• ECSS-E-10-03 Space Engineering - Testing
THE END!

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ESA/ESTEC, Structures Section, NL, provided the data concerning ARIANE 5 FE model and LV/SC CLA
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